Optimization of Hybrid Dealiasing Convolutions

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Overview

- The convolution theorem provides a method for efficiently computing convolutions using Fast Fourier Transforms (FFTs), with the caveat that the convolution is periodic. For two arrays *f* and *g* that have period *M* the convolution is defined as

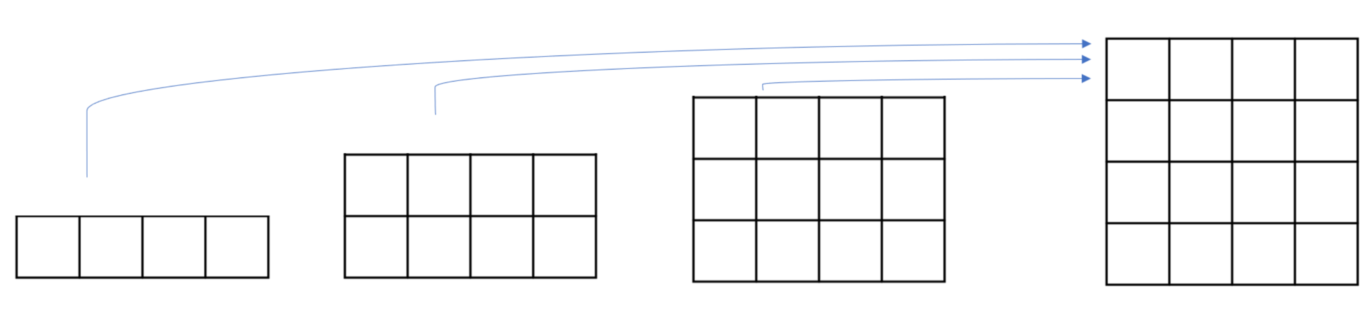
$$(f * g)_l = \sum_{j=0}^{l} f_j \, g_{l-j} \; \forall l \in \{0, \ldots, M-1\}$$

- Many applications such as nonlinear PDEs require a linear convolution, which can be computed using the convolution theorem, provided that the data is sufficiently padded with zeros so we can avoid aliases.
- Implicit dealiasing provides an alternative to explicit dealiasing. To maximize performance, we use hybrid padding: the padding is chosen to be fully implicit, fully explicit, or a combination of the two.

Problem

- Find a rectangle such that I can use its parameters to estimate a large square/rectangles parameters.
- Also create a more optimized and efficient hyperparameter tuning algorithm called *experience* to find the optimal parameters.

Which rectangle's parameters can be used to estimate the square's parameters?

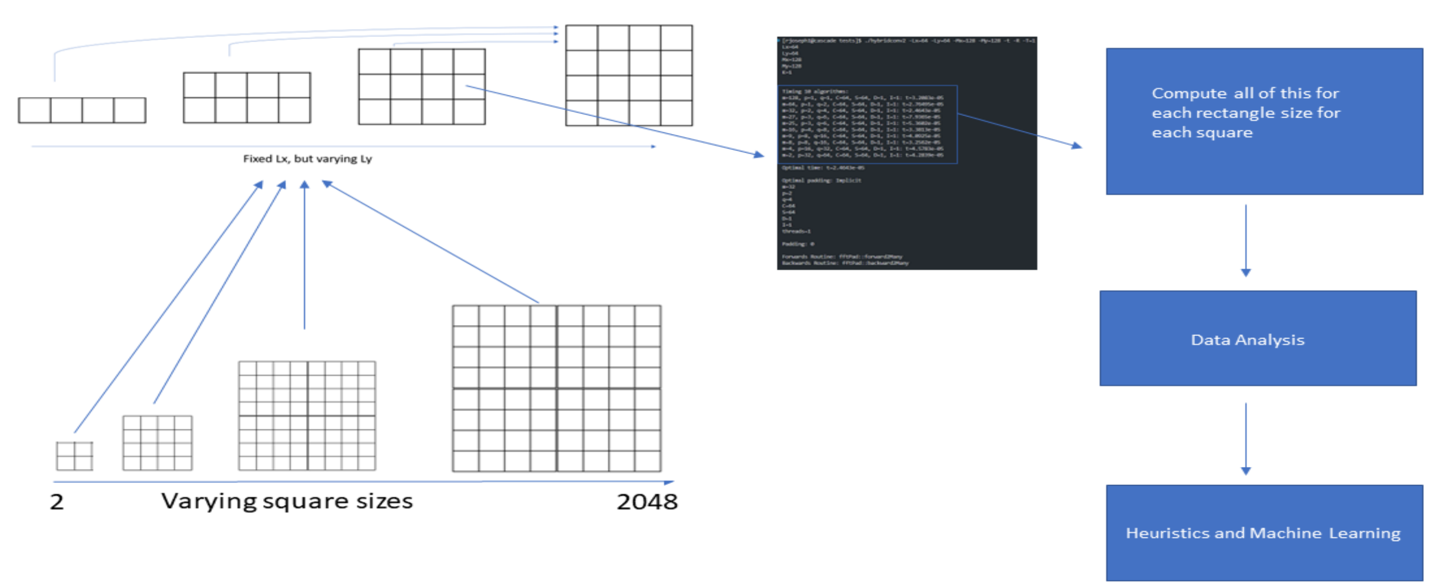


References

- Noel Murasko and John C. Bowman. Hybrid Dealiasing of Complex Convolutions. Submitted *to SIAM J. Sci. Computer*, 2023.

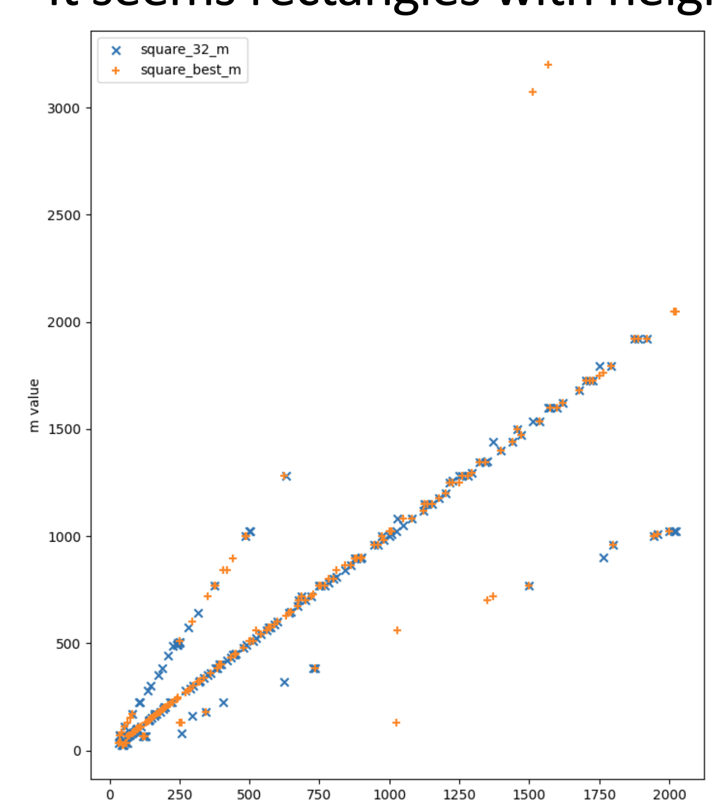
Algorithm

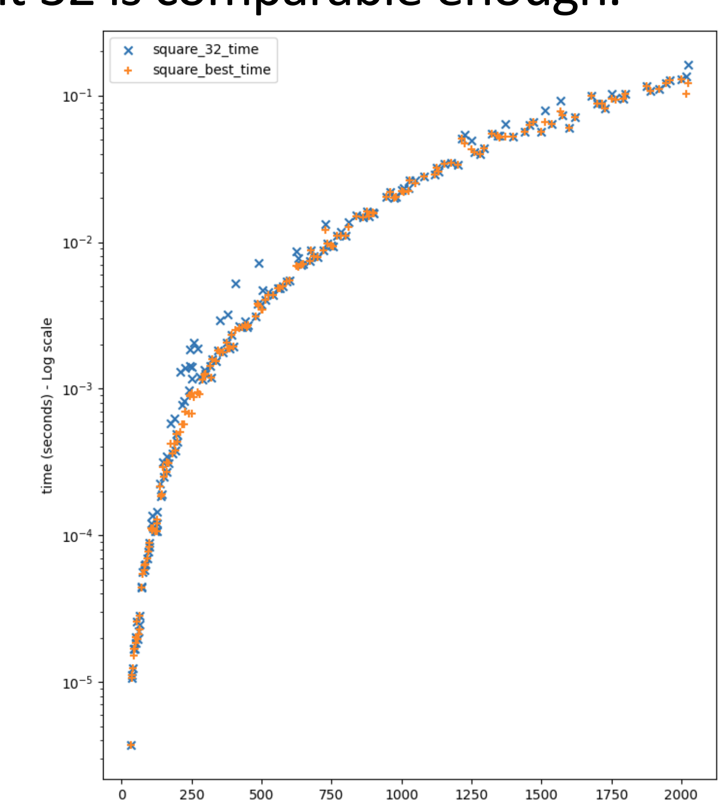
- We generated the data by looking only at square sizes of the form $2^a 3^b 5^c 7^d$ where *a, b, c, d* are integers. For each square we look over all the rectangles by varying the height (Ly) but keeping the same length (Lx).
- We then use the optimizer to get the results for each rectangle and cache all the data.
- Lastly we visualize and analyze the data to check for any patterns, fit Machine Learning models and come up with heuristics. Lastly we evaluate them on other datasets.



Results

- **Takeaway result 1**: 3 interesting rays of *m* values. Only need to search over those 3 rays.
- **Takeaway result 2:** Simply use very *skinny* rectangles instead. It seems rectangles with height 32 is comparable enough.



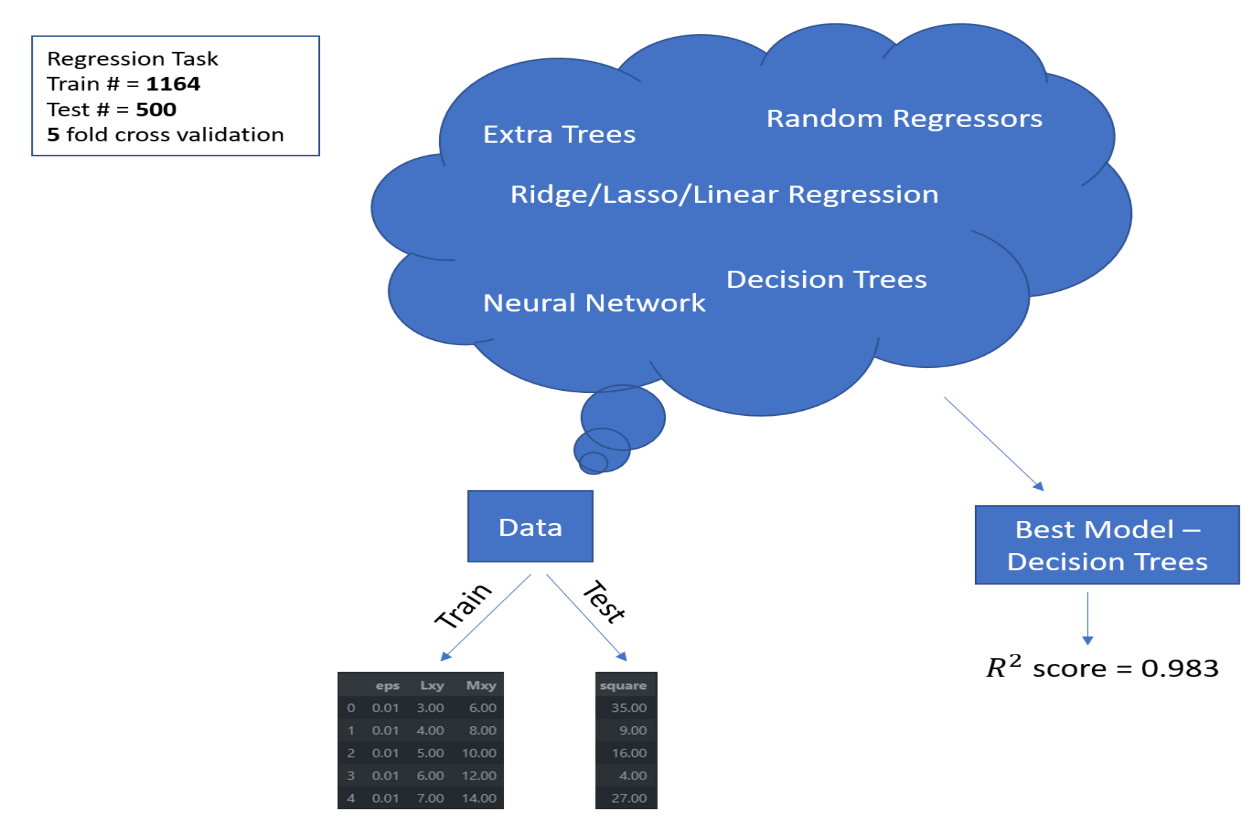


Tuning Parameters

- 3 data-dependent parameters. *L, M* and the number *C* of FFTs to be computed simultaneously.
- Other important parameters. Size *m* of the FFTs, The number *D* of residues at a time, In-place (*I* = 1) or out-of-place (*I* = 0) FFTs.

Machine Learning

- Results seem to be promising especially when switching to a classification task.
- Caveats include that the results are hardware dependent and does not generalize well.
- Conjecture that the predicted value (from classification) timing will be comparable to the ground truth.



Future Work & Applications

- These heuristics will be useful in practice and can contribute to the success of the proposed hybrid dealiasing algorithm.
- We expect hybrid dealiasing to become the standard method for convolutions and are developing hybrid dealiasing for real convolutions and for Imaging and Convolution Neural Networks.